Principles of Nonlinear Optics and selected Applications

Christian Kolleck
Outline

• Motivation
• Basic equations
• Linear and nonlinear effects in optical fibers
• Supercontinuum generation
• Second-order effect: THz-wave generation via optical rectification
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White Light Sources

Coen et al., JOSA B 19, 2002

blazephotonics, UK / Crystal Fibre, Denmark

Fiber ASE source
PCF based super-continuum source
SLEDs (four wavelengths)
Incandescent lamp
**Supercontinuum Generation in Photonic Crystal Fibers**

**Properties** of PCF-generated SC:
- High spatial coherence
- Low temporal coherence

**Applications**:
- Low-coherence interferometry,
- Optical coherence tomography
- Lidar
- Metrology

**DLR, Inst. f. Physik d. Atmosph.**
Applications of Supercontinua: Frequency Metrology

- Exact measurement of frequencies with (known) frequency combs
- Reference frequency: \( \nu_n = n f_{rep} + f_0 \)
- Beating between \( \nu_n \) and unknown \( f_u \):
  \( f_1 = f_u - (n f_{rep} + f_0) \)
- Beating between \( \nu_{2n} \) and 2\(^{nd}\) harmonic \( 2f_u \):
  \( f_2 = 2f_u - (2n f_{rep} + f_0) \)
- Beating between \( f_1, f_2 \):
  \( f_2 - f_1 = f_u - n f_{rep} \)

\[ E(t) \]
\[ I(\nu) \]

Cundiff, Rev. of Mod. Phys. 75, 2003
Terahertz Waves

- Frequency Range: 100 GHz ... >10 THz
- "Technological Gap"

Applications
- Spectroscopy
- Imaging systems for:
  - Medicine
  - Industrial Inspection
  - Security Technology
- Communications

Example THz images of Tissue with Melanome

Security Screening
IEE Review, Dec. 05
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Derivation of propagation equations for optical waves

- Electrical Polarization in nonlinear materials (in the frequency domain):

  \[ P(\omega) = P(1)(\omega) + P(2)(\omega) + P(3)(\omega) + \ldots \]

  \[ P(1)(\omega) = \varepsilon_0 \chi(1) E(\omega) \]

  \[ P(2)(\omega) = \varepsilon_0 \frac{1}{2\pi} \int_{-\infty}^{\infty} \chi(2)(\omega; \omega', \omega - \omega') E(\omega') E(\omega - \omega') d\omega' \]

  \[ P(3)(\omega) = \varepsilon_0 \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi(3)(\omega; \omega', \omega'', \omega - \omega' - \omega'') E(\omega') E(\omega'') E(\omega - \omega' - \omega'') d\omega' d\omega'' \]

- Centrosymmetric Materials (e.g. Glass): \( P^{(2)} = 0 \)
- Non-centrosymmetric Materials (e.g. LiNbO\(_3\), ZnTe): \( P^{(2)} \neq 0 \)
Derivation of propagation equations for optical waves

- Several optical waves, electric field of the \( i \)-th wave:
  \[
  E_i(r, t) = \frac{1}{2} A_i(z, t) \mathcal{E}_i(r, \omega_i) e^{j\omega_i t - \beta_i,0 z} + \text{c.c.}
  \]

  - Slowly varying amplitude
  - Propagator
  - Transverse mode field distribution
  - Complex conjugate

- Expansion of the phase constant \( \beta(\omega) \) around center frequency \( \omega_i \):
  \[
  \beta_i(\omega) = \beta_{i,0} + \beta_{i,1}(\omega - \omega_i) + \frac{1}{2} \beta_{i,2}(\omega - \omega_i)^2 + \frac{1}{6} \beta_{i,3}(\omega - \omega_i)^3 + \ldots
  \]

  \[
  \beta_{i,n} = \frac{\partial^n}{\partial \omega^n} \beta_i(\omega_i) \quad v_{g,i} = \frac{1}{\beta_{i,1}} \quad \text{Group velocity}
  \]

- Inserting into Maxwell’s equations
  \[
  \nabla \times H_0(r, t) = \varepsilon_0 \varepsilon \frac{\partial}{\partial t} E_0(r, t) + \frac{\partial}{\partial t} P^{(3)}_0(r, t)
  \]

  \[
  \nabla \times E_0(r, t) = -\mu_0 \frac{\partial}{\partial t} H_0(r, t)
  \]

  yields propagation equation for amplitudes \( A_i \) (next slide)
Propagation Equation for centro-symmetric Materials
(Coupled Mode Theory)

\[
\frac{\partial}{\partial z}A_1 + \frac{1}{v_g} \frac{\partial}{\partial t}A_1 = -\frac{\alpha}{2}A_1 - \sum_{n=2}^{N} \frac{1}{j^{n-1}n!\beta_n} \frac{\partial^n}{\partial t^n}A_1
\]

Transformation:

\[T = t - z/v_g, \quad Z = z\]

\[\Rightarrow \text{temporal derivative on LHS disappears}\]
Propagation Equation for centro-symmetric Materials
(Coupled Mode Theory)

\[
\frac{\partial}{\partial Z} A_1 = -\frac{\alpha}{2} A_1 - \sum_{n=2}^{N} \frac{1}{j^{n-1}n!} \beta_n \frac{\partial^n}{\partial T^n} A_1
\]

\[
-\left( j\omega_1 + \frac{\partial}{\partial T} \right) \left[ \gamma_{11}|A_1|^2 A_1 \right]
\]

\[
-2 \left( j\omega_1 + \frac{\partial}{\partial T} \right) \left[ \left( \gamma_{12}|A_2|^2 + \gamma_{13}|A_3|^2 + \gamma_{14}|A_4|^2 \right) A_1 \right]
\]

\[
-2 \left( j\omega_1 + \frac{\partial}{\partial T} \right) \left[ \gamma_{1234} A_2^* A_3 A_4 e^{j\Delta k z} \right]
\]

\[
-j\omega_1 \gamma T_R A_1 \frac{\partial |A|^2}{\partial T}
\]

Transformation:
\[
T = t - z/v_g,
\]
\[
Z = z
\]

⇒ temporal derivative on LHS disappears
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\[ \frac{\partial}{\partial Z} A = -\frac{\alpha}{2} A \]

\[ \Rightarrow A(Z, T) = A(0, T) e^{-\alpha Z/2} \]
Group Velocity Dispersion (GVD)

- Second Order Dispersion
  \[ \frac{\partial A}{\partial Z} = j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} \]
- Solution with Fourier Transform
  \[ \frac{\partial}{\partial Z} A = -j \omega^2 \frac{\beta_2}{2} A \]

\[ A(Z, T) = \mathcal{F}^{-1} \left\{ A(0, \omega) e^{-j \frac{\beta_2}{2} \omega^2 z} \right\} \]

- Spectral Bandwidth remains constant
- Additional Phase (quadratically in \( \omega \))
- Temporal Pulse Broadening

Skryabin et al., Science 301, 2003
Group Velocity Dispersion (GVD)

- Example: Gaussian Pulse
  \[ A(0, T) = e^{-T^2/2T_0^2} \Rightarrow A(Z, T) = \frac{T_0}{\sqrt{T_0^4 + j\beta_2 Z}} e^{-\frac{T^2 T_0^2 - j\beta_2 Z}{2 T_0^4 + \beta_2^2 Z^2}} \]

- Instantaneous Phase
  \[ \phi = -\frac{1}{2} \arctan \left( \frac{\beta_2 Z}{T_0^2} \right) + \frac{T^2}{2 \left( T_0^4 + \beta_2^2 Z^2 \right)} \frac{\beta_2 Z}{T_0^4 + \beta_2^2 Z^2} \]

- Instantaneous Frequency
  \[ \delta\omega = \frac{\partial}{\partial T} \phi = \frac{T \beta_2 Z}{T_0^4 + \beta_2^2 Z^2} \]

Linear Frequency Chirp $\delta\omega(Z, T)$
Group Velocity Dispersion (GVD)

- Sign of frequency chirp depends on sign of $\beta_2$:
  - For linearly chirped input pulses: pulse compression possible

![Graph showing blue and red shifts with $\beta_2 > 0$ and $\beta_2 < 0$]

- $\delta\omega$ ($\beta_2 > 0$, Normal dispersion)
- $\delta\omega$ ($\beta_2 < 0$, Anomalous dispersion)

- For linearly chirped input pulses: pulse compression possible
Higher Order Dispersion (HOD)

- Higher-order dispersion terms in propagation equation

\[ \frac{\partial A}{\partial Z} = j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} + \cdots \]

- Solution:

\[ A(Z,T) = \mathcal{F}^{-1} \left\{ A(0,\omega) \exp \left[ \left( -j \frac{\beta_2}{2} \omega^2 - j \frac{\beta_3}{6} \omega^3 - \cdots \right) Z \right] \right\} \]

- \( \Rightarrow \) Strong, asymmetric pulse broadening

*Agrawal: Nonlinear Fiber Optics, Academic Press*
Self Phase Modulation (SPM)

- Third-order nonlinearity in propagation equation:
  \[ \frac{\partial A}{\partial Z} = -j\omega_1 \gamma |A|^2 A \]

- Solution:
  \[ A(Z,T) = A(0,T)e^{j\phi} \]

- Instantaneous nonlinear phase shift:
  \[ \phi(Z,T) = -\omega_1 \gamma |A|^2 Z \]

- Instantaneous frequency chirp:
  \[ \delta\omega(Z,T) = \frac{\partial \phi}{\partial T} = -\omega_1 \gamma \frac{\partial |A|^2}{\partial T} Z \]

  - ⇒ Absolute value of amplitude function remains constant,
  - ⇒ Phase proportional to input power, increases linearly with z,
  - ⇒ Spectral broadening due to phase/frequency shift

- Note: usual definition of \( \gamma \) contains \( \omega \):
  \[ \frac{\partial A}{\partial Z} = -j\gamma |A|^2 A \]
Self Phase Modulation (SPM)

- Example: Gaussian Pulse

\[ A(0, T) = \sqrt{P_0} e^{-T^2/2T_0^2} \Rightarrow |A(Z,T)| = |A(0, T)|, \]
\[ \phi(Z,T) = -\omega_1 \gamma |A(0, T)|^2 Z, \]
\[ \delta\omega(Z,T) = -\frac{T}{T_0^2} \phi(Z,T) \]

- Approx. linear frequency chirp in central region of Gaussian pulse
- \( \gamma \) positive for glass (fibers)

\[ Z \omega_1 \gamma P_0 = 2; 4; 6; 8; 10 \]
Combination of SPM, GVD

- **Propagation equation** (NLSE – nonlinear Schrödinger equation):
  \[
  \frac{\partial A}{\partial Z} = j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - j \omega_1 \gamma |A|^2 A
  \]

- Temporal and spectral form strongly dependent on sign of \( \gamma/\beta_2 \): (\( \gamma > 0 \) in silica fibers)
  - **Normal dispersion** (\( \beta_2 > 0 \)):
    - Temporal spreading can increase (compared to GVD alone),
    - Spectral broadening can decrease (compared to SPM alone)
  - **Anomalous dispersion** (\( \beta_2 < 0 \)):
    - Effects of GVD and SPM can cancel each other out in central region of Pulse
    - **Soliton formation** possible
SPM + GVD: 1. $\beta_2 > 0$ (Normal dispersion)

\[ |A(Z,T)| \]

\[ Z \omega_1 \gamma P_0 = 5 \]

\[ |A(Z,f)| \]

- $\beta_2 = 0, \omega \gamma = 0$
- $\beta_2 = 0, \omega \gamma > 0$
- $\beta_2 > 0, \omega \gamma = 0$
- $\beta_2 > 0, \omega \gamma > 0$

$\frac{T}{T_0}$

$\frac{f}{T_0}$
SPM + GVD: 2. $\beta_2 < 0$ (Anomalous dispersion)

$Z \omega_1\gamma P_0 = 5$

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**SPM + anomalous GVD: Solitons**

- Soliton formation possible
- Order of soliton depending on parameter \( N^2 = (\omega \gamma P_0 T_0^2) / |\beta_2| \)
- First order soliton: \( A(Z,T) \propto \text{sech}(T) \)

Example: Evolution of first order soliton;

Pulse input at \( Z = 0 \):
Gaussian Pulse

\[
A(Z,T) = P_0 e^{-T^2/(2 T_0^2)}
\]
SPM + anomalous GVD: Solitons

Example: Third-Order Soliton

- Periodical behavior!

Further soliton effects:
- Fission of Solitons:
  distorted higher-order solitons split to 1st-order solitons
- Nonsolitonic radiation (NSR):
  Generation of new frequencies (Anti-Stokes)
Self-Steepening

- Inclusion of time derivative in propagation equation:
  \[
  \frac{\partial A}{\partial Z} = -\gamma \left( j\omega_1 + \frac{\partial}{\partial T} \right) (|A|^2 A)
  \]

- Asymmetric spectral broadening

Example:
Evolution of a Gaussian Pulse

\[ A(Z, T) = P_0 e^{-T^2/(2\ T_0^2)} \]
Cross Phase Modulation (XPM)

- Coupling between two or more waves (different frequencies or modes)

\[
\frac{\partial A}{\partial z} + \frac{1}{v_A} \frac{\partial A}{\partial t} = -j \omega_A \left( \gamma_A |A|^2 + 2 \gamma_x |B|^2 \right) A \\
\frac{\partial B}{\partial z} + \frac{1}{v_B} \frac{\partial B}{\partial t} = -j \omega_B \left( \gamma_B |B|^2 + 2 \gamma_x |A|^2 \right) B
\]

- Solution:

\[
A(z,t) = A(0, t - z/v_A) \exp^{j \phi_A} \\
B(z,t) = B(0, t - z/v_B) \exp^{j \phi_B}
\]

\[
\phi_A(z, t) = -\omega_A \left( \gamma_A |A(0, t - z/v_A)|^2 Z + 2 \gamma_x \int_{z'=0}^{z} |B(0, t + \delta z' - z/v_A)|^2 dz' \right)
\]

\[
\phi_B(z, t) = -\omega_B \left( \gamma_B |B(0, t - z/v_B)|^2 Z + 2 \gamma_x \int_{z'=0}^{z} |A(0, t - \delta z' - z/v_B)|^2 dz' \right)
\]

with walk-off parameter \( \delta = \frac{1}{v_A} - \frac{1}{v_B} \)
Cross Phase Modulation (XPM)

- Two or more waves induce nonlinear phase shift to each other
- Efficiency dependent on walk-off $\delta$ and pulse duration $T_0$
- Spectral broadening dependent on temporal overlap

**Case: $\delta = 0$** (equal group velocities $v_A = v_B = v$):

\[
\phi_A(z, t) = -\omega_A \left( \gamma_A |A|^2 + 2\gamma_x |B|^2 \right) z
\]
\[
\phi_B(z, t) = -\omega_B \left( \gamma_B |B|^2 + 2\gamma_x |A|^2 \right) z
\]

⇒ Cross-phase shift proportional to Power of the other wave

**Case: $\delta L > T_0$**, one pulse passes the other

\[
\phi_A = -\omega_A \left( \gamma_A |A|^2 z + 2\gamma_x \frac{1}{\delta} \int_{t=-\infty}^{\infty} |B|^2 dt \right)
\]
\[
\phi_B = -\omega_B \left( \gamma_B |B|^2 z + 2\gamma_x \frac{1}{\delta} \int_{t=-\infty}^{\infty} |A|^2 dt \right)
\]

⇒ Contribution of XPM results in constant phase shift
**Frequency Mixing Processes**

- **Sum frequency generation (SFG):**
  
  Generation of $\omega_4$ with
  
  $$\omega_4 = \omega_1 + \omega_2 + \omega_3$$

  if frequencies $\omega_i$, $i=1, 2, 3$ are present

- Equation for $A_4$:
  
  $$\frac{\partial A_4}{\partial z} = -2j \omega_4 \gamma A_1 A_2 A_3 e^{j \Delta k z}$$

- Difference of the phase constants:
  
  $$\Delta k = \beta(\omega_4) - \beta(\omega_1) - \beta(\omega_2) - \beta(\omega_3)$$

- Condition for efficient SFG: $\Delta k = 0$

\[P_4 \propto |A_4|^2\]

for Third harmonic generation

\[P_4 / \text{a.u.}\]

\[\Delta k = 0\]

\[\Delta k \neq 0\]
**Frequency Mixing Processes**

- **Four Wave Mixing (FWM)**
  Nonlinear interaction of 4 waves
  \[ \omega_1 + \omega_2 = \omega_3 + \omega_4 \]
  At least 3 of these frequencies have to be present to produce the 4th frequency

- Equation for amplitude \( A_1 \):
  \[
  \frac{\partial A_1}{\partial z} = -j \omega_1 \left[ \left( \gamma_{11} |A_1|^2 + 2 \sum_{k<>1} \gamma_{1k} |A_k|^2 \right) A_1 + 2 \gamma_x A_2^* A_3 A_4 e^{j \Delta k z} \right]
  \]
  Similar equations for \( A_2, A_3, A_4 \)

- Difference of the phase constants:
  \[ \Delta k = \beta(\omega_1) + \beta(\omega_2) - \beta(\omega_3) - \beta(\omega_4) \]

- Condition for efficient interaction (for low input powers):
  \[ \Delta k = 0 \]
**Frequency Mixing Processes**

- **Four Wave Mixing (FWM), degenerate case** $\omega_1 = \omega_2$
  Nonlinear interaction of 3 waves

  $$2\omega_1 = \omega_3 + \omega_4$$

- Typical application: Amplification of signal wave $\omega_s$ through a strong pump $\omega_p$, while generating an idler wave $\omega_i$

- Phase matching condition (for low input powers)

  $$2\beta(\omega_1) = \beta(\omega_3) + \beta(\omega_4)$$
**Frequency Mixing Processes**

- **Four Wave Mixing (FWM), degenerate case** $\omega_1 = \omega_2$

  Coupled equations for non-depleted pump and $|A_1| \gg |A_3|, |A_4|

  \[
  A_1(z) = \sqrt{P_1} e^{-2j\omega_1 \gamma_1 P_1}
  \]

  \[
  \begin{align*}
  \frac{\partial A_3}{\partial z} &= -j\omega_3 \left( 2\gamma P_1 A_3 + \gamma_x A_1^2 A_4^* e^{j\Delta k z} \right) \\
  \frac{\partial A_4}{\partial z} &= -j\omega_4 \left( 2\gamma P_1 A_4 + \gamma_x A_1^2 A_3^* e^{j\Delta k z} \right)
  \end{align*}
  \]

- **Solution:**

  \[
  \begin{align*}
  A_3 &= \left( a_3 e^{g_z} + b_3 e^{-g_z} \right) e^{j\kappa z/2} \\
  A_4 &= \left( a_4 e^{g_z} + b_4 e^{-g_z} \right) e^{j\kappa z/2}
  \end{align*}
  \]

- **Parametric gain:**

  \[
  g = \sqrt{\Gamma P_1 \Delta k - (\Delta k/2)^2}
  \]

- $\Rightarrow$ maximum gain for $\Delta k = 2 \Gamma P_1$
Phase Matching in standard single mode fibers.
- a) all waves are the same mode
- b) different mode for AS

(Osborne, Opt. Lett. 19, 1994)

Phase Matching in PCF.

(Dudley, JOSA B 19, 2002)

In most cases, the pump has to be near the zero dispersion wavelength (ZDW).

ZDW = 1300 nm  
ZDW = 765 nm
Stimulated Raman Scattering (SRS)

- Generation of new frequencies, essentially waves on the Stokes side (lower frequencies)
- Evolution of the Intensities
  \[ I_p = \text{Intensity of Pump}, \]
  \[ I_s = \text{Intensity of Stokes wave} \]

\[
\frac{\partial I_s}{\partial z} = g_R I_p I_s, \\
\frac{\partial I_p}{\partial z} = -\frac{\omega_p}{\omega_s} g_R I_p I_s
\]

SRE = Spontaneous Raman Effect
IRE = Induced Raman Effect (=SRS)
Stimulated Raman Scattering (SRS)

- Pulse propagation: Consideration of Pump and Stokes as ONE wave; delayed Raman response; temporal behavior:

\[
\frac{\partial A}{\partial Z} = \ldots - \left( j \omega_1 + \frac{\partial}{\partial T} \right) \left[ A \int_{-\infty}^{\infty} R(T') |A(Z, T - T')|^2 dT' \right]
\]

\[R(T') = (1 - f_R)\delta(T) + f_R h_R(T)
\]

Delayed Raman response function \( h_R(T) \)

Stolen et al., JOSA B 6, 1989
**Soliton Self Frequency Shift (SSFS)**

- **Solitons** in combination with **SRS**:  
  ⇒ continuous frequency downshift of the soliton

*Liu et al., Opt. Lett. 26, 2001*
**Non-Solitonic Radiation (NSR)**

- **Solitons** in combination with HOD:
  \[ \Rightarrow \text{continuous frequency downshift of the soliton} \]
- At the same time: emission of **blue-shifted radiation** at frequency which is **phase-matched** to soliton

Nonlinear effects: Summary

- Broadening or Shift of spectrum:
  SPM, XPM, SS, SRS, SSFS, NSR
- Generation of new frequencies: SRS, FWM, SFG, NSR
- CW effects: SPM, XPM, SRS, FWM, SFG
- ps-pulse effects: CW effects + Soliton effects
- fs-pulse effects: ps effects + SS
- Phase matching required: FWM, SFG, NSR
- Velocity matching required: FWM, SRS, SFG, XPM
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Historical Development of SC-Generation Experiments
(according to Coen et al., JOSA B 19, 2002)

• 1st Generation: Bulk materials with $\chi^{(3)}$-nonlinearity
  • Primary Mechanism: SPM
  • Example: Continuum from 80fs-pulses in KDP (Fork et al., Opt. Lett. 8, 1983)

• 2nd Generation: Spectral Broadening of ps-pulses in silica fibers
  • Longer interaction lengths, higher intensity
  • Reduction of required pump power
  • Primary effects: SRS, FWM
  • Example: Continuum from 30ps@552µm-pulses in 15m silica fiber (Baldeck et al., JLT 15, 1987)
  • Simulations: Continuum from 33,3ps@1319µm-pulses in silica fiber (Osborne, Opt. Lett. 19, 1994)
Historical Development of SC-Generation Experiments

- **3rd Generation**: Spectral Broadening of ps-pulses in silica fibers
- Focus on SC generation with GHz repetition rates and noise characteristics for possible applications in WDM systems
- Primary effects: SPM and GVD (normal, anomalous)

Example: Examination of the phase coherence. Slices of the continuum from 3.5ps@1540μm-pulses in silica fibers with 10GHz repetition rate and various dispersion profiles

(Tamura et al., JQE 36, 2000)
Historical Development of SC-Generation Experiments

- **4th Generation**: Spectral Broadening of ps-pulses in Photonic Crystal Fibers

Advantages of PCF:

- High index contrast ⇒ High confinement of modes possible ⇒ Higher Intensities

- Tailoring the geometry ⇒ Adjustment of GVD characteristics possible ⇒ e.g. Zero dispersion at 800nm ⇒ Pump can travel long distances without temporal spreading

Knight et al., Photon. Tech. L. 12, 2000

Genty et al., Opt. Expr. 12, 2004
SC-Generation in PCF

- Modeling:

\[
\frac{\partial A}{\partial Z} = -\frac{\alpha}{2} - \sum_{n=2}^{N} \frac{1}{n!} \beta_n \frac{\partial^n A}{\partial T^n} - \left(j \omega_1 + \frac{\partial}{\partial T}\right) \left[ A \int_{-\infty}^{\infty} R(T') |A(Z, T - T')|^2 \, dT' \right]
\]

- Different Mechanisms for pumping in the anomalous or normal dispersion regime

**Genty et al., Opt. Expr. 10, 2002**

**Hundertmark et al., Opt. Expr. 11, 2003**
SC-Generation in PCF: Pump in normal dispersion regime

- Mechanism:
  - SPM
  - SRS
  - SS
  - FWM
  - SSFS

Genty et al., Opt. Expr. 10, 2002

Dudley et al., Opt. Expr. 10, 2002
SC-Generation in PCF: Pump in anomalous dispersion regime

- Mechanism:
- Pulse Compression by NL, Disp.
- Multiple Soliton break-up
- SSFS -> red shift
- Non-Solitonic radiation -> blue frequencies
- FWM

Genty et al., Opt. Expr. 10, 2002
SC-Generation in PCF: Pump in anomalous dispersion regime

- Small amplitude, rather long initial pulse duration
- Mechanism:
- Higher order soliton of Nth order
- Fission of soliton into N 1st order solitons because of HOD
- Frequency shift of solitons and NSR

Herrmann et al., PRL 88, 2002
Conclusion

- SC-generation based on combination of linear and nonlinear effects: GVD, HOD, SPM, XPM, SS, SRS, FWM, SSFS, NSR
- Phase matching and velocity matching necessary
- Characteristic of SC strongly dependent on GVD ⇒ PCF
- SC generation in PCF possible with
  - High intensities,
  - Spectrum of 2 octaves or above
  - Fixed phase relationship of output (⇒ frequency combs)
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Second-order Nonlinearities

- Electrical Field 
  \[ \mathbf{E}(t) = \mathbf{E}_0 + \frac{1}{2} \sum_n \mathbf{E}_n e^{j\omega nt} \]

- Linear Polarization 
  \[ \hat{P}^{(1)} = \varepsilon_0 \chi^{(1)} \mathbf{E}_n \]

- Nonlinear Polarization 
  \[ \hat{P}^{(2)} = \frac{\varepsilon_0}{4\pi} \chi^{(2)} (\omega_p + \omega_q; \omega_p, \omega_q) \mathbf{E}_p \mathbf{E}_q \]

- Second-order nonlinear effects:

<table>
<thead>
<tr>
<th>( \omega_p )</th>
<th>( \omega_q )</th>
<th>( \omega_p + \omega_q )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 0 )</td>
<td>( \geq 0 )</td>
<td>( \omega_p + \omega_q )</td>
<td>Sum Frequency Generation (SFG)</td>
</tr>
<tr>
<td>( \geq 0 )</td>
<td>= ( \omega_p )</td>
<td>2 ( \omega_p )</td>
<td>Second Harmonic Generation (SHG)</td>
</tr>
<tr>
<td>( \geq 0 )</td>
<td>( \leq 0 )</td>
<td>( \pm (</td>
<td>\omega_p</td>
</tr>
<tr>
<td>( \geq 0 )</td>
<td>- ( \frac{1}{2} \omega_p )</td>
<td>( \frac{1}{2} \omega_p )</td>
<td>Degenerate DFG</td>
</tr>
<tr>
<td>( \geq 0 )</td>
<td>- ( \omega_p )</td>
<td>0</td>
<td>Optical Rectification (OR)</td>
</tr>
<tr>
<td>( \geq 0 )</td>
<td>0</td>
<td>( \omega_p )</td>
<td>Linear Electrooptic (EO) Effect</td>
</tr>
</tbody>
</table>
Mathematical description of rectified field

- Optical waves
  \[ E_i(r, t) = \frac{1}{2} A_i(z, t) \mathcal{E}_i(r_t, \omega_i) e^{j\omega_t - \beta_i,0 z} + c.c. \]

- Rectified field:
  1. Ansatz with amplitude function (plane waves, guided waves)
     \[ E_{t,0}(r, t) = \sum_m A_{m,0}(z, t) \mathcal{E}_{tm}(r_t, \omega \to 0) \]
  2. or as solution of Maxwell's equations (THz radiation in multiple directions)

\[ \nabla \times H_0(r, t) = \varepsilon_0 \varepsilon \frac{\partial}{\partial t} E_0(r, t) + \frac{\partial}{\partial t} P_0^{(2)}(r, t), \quad \nabla \times E_0(r, t) = -\mu_0 \frac{\partial}{\partial t} H_0(r, t) \]

(Time-dependent fields)

\[ \nabla \times E_0(r) = 0, \quad \nabla \cdot \left( \varepsilon_0 \varepsilon E_0(r) + P_0^{(2)}(r) \right) = 0 \]

(Static fields)

⇒ Coupled (P)DEs for \( A_{mq}(z,t) \) and \( A_{m0}(z,t) \) or \( E_0(r,t) \)
Optical rectification (plane waves)

- Amplitude of optical fs pulse
  \[ A(z, t) = A(t - \beta_A z) \]
- Neglecting Losses and Dispersion: Amplitude of rectified field \( B(z, t) = B_+(z, t) + B_-(z, t) \) as solution of
  \[
  \left( \frac{\partial}{\partial z} + \beta_B \frac{\partial}{\partial t} \right) B_+(z, t) = -\kappa(2) \frac{\partial}{\partial t} |A(z, t)|^2, \\
  \left( \frac{\partial}{\partial z} - \beta_B \frac{\partial}{\partial t} \right) B_-(z, t) = \kappa(2) \frac{\partial}{\partial t} |A(z, t)|^2
  \]
  \( \Rightarrow \) Two THz pulses in forward and backward direction
- Amplitude and temporal separation of THz pulses depending on \( d = \beta_B - \beta_A \)
Optical rectification (plane waves)

- Amplitude of optical fs pulse
  \[ A(z, t) = A(t - \beta_A z) \]

- Neglecting Losses and Dispersion: Amplitude of rectified field \( B(z, t) = B_+(z, t) + B_-(z, t) \) as solution of
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  \end{align*}
  \]

  \( \Rightarrow \) Two THz pulses in forward and backward direction

- Amplitude and temporal separation of THz pulses depending on \( d = \beta_B - \beta_A \)
Optical rectification (plane waves) in periodically poled material

Periodical poling of nonlinear material (e.g. LiNbO$_3$)
- Concatenation of single pulses generated in domain interfaces
  - $\Rightarrow$ Narrowband THz-Field
- Center frequency given by walk-off between optical and THz pulses in one domain
  \[ \omega_m = \frac{\pi}{\tau_{\pm}} = \frac{\pi}{|\beta_B \mp \beta_A| L} \]
- Bandwidth given by Number $N$ of domains

Measured THz field
Lee et al., APL 76, pp. 2505 (2000)
**THz fields in 2D-Geometries**

- Inhomogeneous (2+1)-dimensional wave equation
  \[
  \left( \frac{1}{\xi^2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \psi = -Q
  \]

- Nonlinear polarization:
  \[P_0(x, z, t) = p(x)|A(t - \beta_A z)|^2\]

- In homogeneous space: Solution with Green’s function of moving source
  - Two bipolar pulses in every direction, generated at domain boundaries
  - Separation of pulses depending on direction

\[\frac{t}{t_0} = 21.0\]

**Cherenkov angle \(\alpha\):**
\[\cos \alpha = \frac{c}{v_{g, opt}}\]

\[E_{y0}(h/2, z) / \text{[mV/m]} \quad \text{mV/m} \quad E_{y0} / \text{[mV/m]} \quad \text{Kolleck, Proc. SPIE 5971, Nr. 80 (2005)}\]
\( N \) domains: Superposition of single THz pulses to continuous wave

\[ f(\vartheta) = \frac{1}{2\tau} = \frac{c}{2L(\cos \alpha - \cos \vartheta)} \]

\( \Rightarrow N+1 \) bipolar Pulses in every direction \( \vartheta \)

Center frequency depending on \( \vartheta \)

Electrical field

Power spectrum

Kolleck, Proc. SPIE 5971, Nr. 80 (2005)
Conclusion

• Optical rectification of fs pulses yields THz pulses
• Single-domain material:
  – Two THz pulses with one/few half cycles in each direction
• Periodically poled material:
  – Narrowband THz wave
  – Center frequency depending on walk-off between optical and THz field in one domain
  – Bandwidth depending on number of domains